Original Research

Sensitivity of Beta and Weibull Synthetic Unit Hydrographs to Input Parameter Changes

Andrzej Wałęga[1]*, Agnieszka Rutkowska[2], Agnieszka Policht-Latawiec[3]

[1]Department of Sanitary Engineering and Water Management, University of Agriculture in Kraków, Mickiewicza 24-28, 30-059 Kraków, Poland
[2]Department of Applied Mathematics, University of Agriculture in Kraków, Balicka 253C, 30-198 Kraków, Poland
[3]Department of Land Reclamation and Environmental Development, University of Agriculture in Kraków, Mickiewicza 24-28, 30-059 Kraków, Poland

Received: 11 May 2013
Accepted: 22 December 2013

Abstract

Our article investigates the response of the synthetic unit hydrographs (SUH) to parameter changes using sensitivity analysis. The beta and Weibull probability density functions and the objective function that includes both the peak discharge and time to peak were chosen to construct the SUHs for a small mountain catchment. The absolute and relative dependences between the distribution parameter changes and changes in the peak discharge and time to peak are measured using the sensitivity analysis methods which include both differential calculus and numerical computations. The Weibull SUH turned out to be more sensitive than the beta SUH when modeling the maximum discharge. The impacts of both parameters of the beta SUH and one parameter of the Weibull SUH on the time to peak are comparable, while the influence of the second parameter of the Weibull SUH is weaker. The results of validation showed good efficiency of the models and compatibility with Snyder's model. They serve as an effective tool in assessing flood risk.

Keywords: sensitivity analysis, synthetic unit hydrograph, beta and Weibull distributions, peak discharge, time to peak

Introduction

The European Union Floods Directive requires drawing maps of flood hazard and risk for all river basins according to three scenarios: low, medium, and high hazard. In order to prepare such maps, detailed knowledge of flood wave characteristics, peak discharges, and duration times is required [1-6]. Particular attention must be paid to methods of determining hypothetical hydrographs, which are applied to find flood hazard zones, to design hydraulic structures, and to assess the conditions in a catchment. The knowledge of these waves is also crucial for the transformation of a wave in a watercourse [7-9].

*e-mail: a.walega@ur.krakow.pl

The main aim of our paper is the assessment of the sensitivity of two synthetic unit hydrograph (SUH) models, applied to a small mountain catchment, to parameter changes. Two SUH models described by the two-parameter beta and two-parameter Weibull probability density functions are constructed in the first stage of the analysis. The objective function that includes both the peak discharge and the time to peak is applied. This step is important for applications because, due to a large variety of catchments and climatic conditions that prevail in different world regions, it is necessary to determine a set of model parameters relevant to local conditions. Then the models are assessed using the sensitivity analysis that evaluates the parameter impacts on the model response, selects the most important variables, and investigates the effect of error in input on the error in output. The absolute and relative sensitivity of the maximum discharge and time to peak are considered. The models are validated using the Nash-Sutcliffe coefficient, relative errors, and comparison to the traditional Snyder's model. Generally, we attempted to demonstrate the usefulness of the mentioned SUHs to describe the flood wave of various origins in a medium-sized mountain catchment.

The decision to choose the beta and Weibull probability density functions was based on Bhunya [10], who observed their great shape flexibility to parameter changes when compared to other distribution.

Owing to the fact that the area under a synthetic unit hydrograph is 1, it may be expressed by a properly selected probability density function. The models that use a probabilistic description of the hydrograph are featured usually by a small number of parameters. This simplifies the calibration and further application [11]. Synthetic unit hydrographs are particularly useful to the catchments where no regular hydrometric measurements are conducted [12]. The parameters of the SUH are determined on the basis of catchment characteristics and not, as in traditional unit hydrographs, on the basis of registered precipitation and outflow series [13]. The methods of making synthetic unit hydrographs widely used in practice include: Snyder's method [13, 14], SCS [15], Clark's method [16, 17], and Gray's method [10, 18]. In practice, a proper projection of a unit hydrograph is achieved for lognormal, gamma, beta, Weibull, and chi-square distributions. They were applied, for example, in northern India and Bangladesh and were described by Bhunya et al. [10, 19].

Catchment Description

The study was conducted for a mountain basin of the Jasiolka River located in southeastern Poland. The total area of the catchment is 513 km² and its average elevation is 541.55 m a.s.l. The parameters of the catchment are presented in Table 1 and the hydrographic network and land cover in Fig. 1.

Table 1. Parameters of the Jasiołka catchment.

Index	Value
River network density G_s [km·km^{-2}]	2.28
Maintenance factor S_w [km^2·km^{-1}]	0.44
Lake density W_j [%]	0.03
Average river slope J_o [‰]	7.40
Average width of the catchment B_z [km]	9.55
Length of the catchment L [km]	53.69
Coefficient of form CF [-]	0.18
Elongation coefficient CW [-]	0.48
Gravelius coefficient [-]	2.08

The catchment is located in the outer flysch Carpathians, where the predominant rock type is marly silicate flysch. The main soil types are brown earths, lessive soils, and rendzinas. As can be seen in Fig. 1, the catchment is typically agricultural: agricultural lands cover 53.9% and forests about 40%. The rest of the catchment is covered by built-up areas, orchards, fallow lands, and grasslands.

Material and Methods

The two rainfall-runoff events of summer and winter half-year were applied to the analysis. They represented flood events from the period 1980-83. Due to data availability, the time step was equal to one day. Dimensionless values $\frac{t_i}{b}$ were adopted in the unit hydrographs, where t_i is the time from the beginning of the hydrograph and b is the duration of the unit hydrograph. Before the actual analysis, the baseflow and the direct runoff were separated on a hydrograph by a straight line from a point where the flow starts to increase to a point where the direct runoff ends.

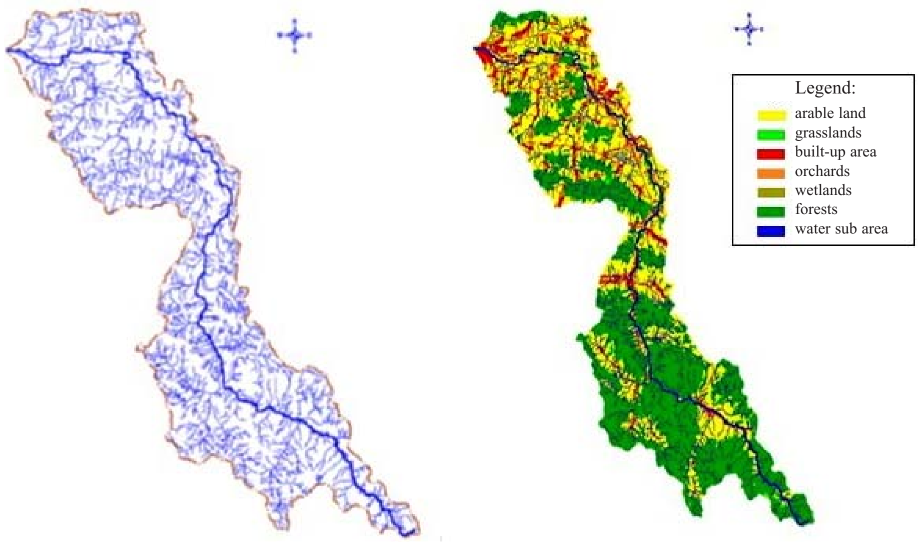

Fig. 1. Hydrographic network and land use in the catchment.

The observed unit hydrograph ordinates were determined using the method described by Ponce [8].

Construction of the SUH

The beta and Weibull distributions were considered for the construction of the synthetic unit hydrographs:

- the two-parameter beta distribution with parameters α, β > 0 and the probability density function in the form:

$$f(t) = \frac{1}{B(\alpha,\beta)} t^{\alpha-1}(1-t)^{\beta-1} \qquad 0 \le t \le 1 \quad (1)$$

...where B is the beta function.

The mode and f(mode) are given as

$$mode = \frac{(\alpha-1)}{\alpha+\beta-2} \text{ and } f(mode) = \frac{(\alpha-1)^{\alpha-1}(\beta-1)^{\beta-1}}{(\alpha+\beta-2)^{\alpha+\beta-2}B(\alpha,\beta)} \quad (2)$$

- the two-parameter Weibull distribution with parameters v>0, k>1 has the probability density function

$$f(t) = \frac{k}{v}\left(\frac{t}{v}\right)^{k-1} e^{-\left(\frac{t}{v}\right)^{k}}, \quad t \ge 0 \qquad (3)$$

The mode and f(mode) are

$$mode = v\left(\frac{k-1}{k}\right)^{\frac{1}{k}} \text{ and } f(mode) = \frac{k}{v}\left(\frac{k-1}{k}\right)^{\frac{k-1}{k}} e^{-\frac{k-1}{k}} \quad (4)$$

The estimators of α, β, k, v were calculated using the optimization method.

Optimization Process

The criterion for optimization was to minimize a scalar quantity known as an objective function or an error. In this study the following objective function for a single event was applied:

$$F = \left[\left(\frac{q_{pobs} - q_{psim}}{q_{psim}}\right)^2 + \left(\frac{T_{pobs} - T_{psim}}{T_{psim}}\right)^2\right]^{1/2} \qquad (5)$$

...where q_{pobs} and q_{psim} are the observed and simulated peak discharges in the SUH and T_{pobs} and T_{psim} are the times to peak in the SUH. For the catchment considered, the winter and summer peak discharges of unit hydrograph were 1.92 $m^3 \cdot s^{-1}$ and 3.44 $m^3 \cdot s^{-1}$, respectively, and the times to peak were 72 h for winter and 48 h for summer. The time to peak is higher for the winter half-year wave than for the summer half-year because, according to Wałęga [20], the flow increase is slower in the analyzed region in the winter half-year. This is caused by supplying the watercourse mainly by the snowmelt runoff. In the summer half-year the runoff comes from heavy and torrential precipitation which is then manifested by a rapid increase over a short time and slow decrease of flows. Change in supply affects the shape of hydrographs.

The function F was introduced by Lee et al. [21] and adopted by Al-Wagdany and Rao [22]. The advantage of using this form of the objective function is that it takes into consideration both peak discharge q_p and time to peak T_p in the SUH. According to Ahmad et al. [23], it is particularly useful in models that aim to determine the flows for design of hydraulic structures and flood protection models. Those authors applied the function to optimize the Clark IUH model for catchments in Pakistan. The function F takes the lowest value equal to 0 if and only if q_{obs}=q_{sim} and T_{pobs}=T_{psim}. As T_{pobs} is the observed time to peak, it can be treated as estimate of the mode of the distribution. Analogously, the peak discharge q_{obs} estimates f(mode). Hence, the parameters α, β, k, v can be estimated by solving the system of equations:

$$T_{pobs} = mode, \quad q_{obs} = f(mode) \qquad (6)$$

These equations are expressed in (2) and (4) for the beta and Weibull distributions. The solutions of (2) and (4) provide estimates of parameters.

Sensitivity of the SUH

Sensitivity is the rate of change in a function Y with respect to change in an argument $X=(X_1, X_2, ..., X_n)$ [24]. The base for nearly all sensitivity analysis techniques is differential calculus. Let $D \subset R^n$ be an open and bound set in a Euclidian measure. Let $Y:D \to R$ be a function of class $C^{(\infty)}$ for every i=1, …, n, $X_0=(X_{10},\ldots,X_{n0}) \in D$ and all partial derivatives $\frac{\delta^m Y}{\delta X_i^m}$ be uniformly bounded on some neighborhood $U \subset D$ of X_0. Assume that all but the variable of interest, X_i, are held fixed and that $(X_{10}, \ldots, X_{i0}+\Delta X_i, \ldots, X_{n0}) \in U$. Then a Taylor series expansion of the function Y at X_0 has the form:

$$Y(X_{10}, \ldots, X_{i0} + \Delta X_i, \ldots, X_{n0}) =$$

$$Y(X_0) + \frac{\delta Y}{\delta X_i}(X_0)\Delta X_i + \cdots + \frac{1}{m!}\frac{\delta^m Y}{\delta X_i^m}(X_0)\Delta X_i^m + \cdots \qquad (7)$$

The contribution of each component to the total sum decreases with m. We limit ourselves to the case when nonlinear terms in the above equation are small in comparison to the linear one. Then the equation reduces to:

$$Y(X_{10}, \ldots, X_{i0} + \Delta X_i, \ldots, X_{n0}) = Y(X_0) + \frac{\delta Y}{\delta X_i}(X_0) \cdot \Delta X_i$$

Hence, the incremental change in Y is:

$$\Delta Y_0 = Y(X_{10}, \ldots, X_{i0} + \Delta X_i, \ldots, X_{n0}) - Y(X_0) = \frac{\delta Y}{\delta X_i}(X_0) \cdot \Delta X_i$$

...and the sensitivity of Y to changes in X_i is expressed in an analytical way as the partial derivative $\frac{\delta Y}{\delta X_i}$. If the partial derivation is featured by a high computational complexity, which happens often in hydrological models, the approximated methods are applied. Then the sensitivity is an approximation of the first partial derivative and in practice is expressed as the ratio of two changes:

$$S = \frac{\Delta Y_0}{\Delta X_i} \qquad (8)$$

This is the first form of sensitivity, which is called *absolute sensitivity*. In fact, it measures the power of the linear part of the *i*-th variable in the function. The second form is the *relative sensitivity,* which expresses the relative change in Y with respect to a relative change in X_i:

$$e = \frac{\Delta Y/Y_0}{\Delta X_i/X_i} = \frac{\%change_{output}}{\%change_{input}} \tag{9}$$

This coefficient is also called the elasticity ratio. These two parameters are used in hydrological modeling [24, 25] and enable us to compare models. The elasticity *e* is invariant to the dimensions of *X* and *Y*. If $|e|\geq 1$, then the parameter is *flexible*. In other words, the dependent variable is sensitive to changes of the independent variable. Otherwise, if $|e|<1$, then the parameter is weak *flexible* or *inflexible* and the dependent variable is weak sensitive or insensitive to changes of the independent variable.

In our study, each of the estimated parameters α, β, v, k was treated as an independent input variable. The point (α_{min}, β_{min}) plays the role of X_0 in (7) and *Y* is the peak discharge or time to peak simulated by the beta SUH. Analogously, we have $X_0=(k_{min}, v_{min})$ for the Weibull SUH. Then their influence on the peak discharge and time to peak was reflected in *S* and in *e*. The time to peak and the peak discharge is expressed in (2) and (4) as *mode* and *f*(*mode*), respectively. Both functions can be expanded in a Taylor's series around (α_{min}, β_{min}) and (k_{min}, v_{min}). Notice that *f*(*mode*) that equals the simulated peak discharge is featured by a high complexity in (2) and (4) (except for dependence $v\rightarrow f(mode)$ in Weibull pdf). Its sensitivity analysis will be based mainly on numerical calculations. The time to peak, i.e. *mode* in (2) and (4), has a simpler form and the absolute sensitivity can be calculated analytically using the partial derivatives. The simplest, linear dependence exists between *v* and *mode* in (4).

The investigation of the maximum discharge variability was also performed when both parameters were changed simultaneously. For some pairs (α, β), from neighborhood of (α_{min}, β_{min}), the simulated peak discharges in beta SUH were calculated. The analogous analysis was performed for the Weibull SUH. This part of analysis also plays an important role in assessing the flood hazard.

Validation of the Models

The models were validated in two ways: with the Nash-Sutcliffe efficiency coefficient and using the comparison to the classical Snyder UH model.

The Nash-Sutcliffe coefficient [26, 27] assesses the quality of the generated SUH. It compares the shape of the obtained pdf to the shape of the observed unit hydrograph:

$$E = 1 - \frac{\sum_{i=1}^{NQ}\left(Q_{i,obs} - Q_{i,sim}\right)^2}{\sum_{i=1}^{NQ}\left(Q_{i,obs} - \overline{Q_{obs}}\right)^2} \tag{10}$$

...where *NQ* is the number of ordinates of the hydrograph, $Q_{i,obs}$ and $Q_{i,sim}$ are the ith ordinates of the observed and simulated hydrograph and $\overline{Q_{obs}}$ is the mean value of the ordinates of the observed hydrograph. In the validation process, the criterion of model quality presented in the paper by Moriasi et al. [28] will be used. It states that, if the efficiency ratio *E* exceeds 65%, the model quality is good and if it exceeds 75%, then very good.

Each SUH model was also compared to the Snyder UH model for the summer half-year wave using the relative error.

The simulations for the Snyder's model were carried out using HEC-HMS 3.4 software [29]. The effective precipitation was determined by the SCS method, at present known as NRCS-CN [16, 30], assuming the value of CN as 82. The baseline flow was determined using the recession curve parameters. NRCS-CN is alternative method to runoff determination, especially in an ungauged catchment [4].

The parameters of the Snyder's model were determined using the methodology given by Ven Te Chow et al. [1] and Ponce [8]. The peak discharge and the lag time were calculated using the formulas:

$$q_p = \frac{2.78 \cdot C_p \cdot A}{T_l}, \qquad T_l = C_t \cdot (LL_c)^{0.3} \tag{11}$$

...where C_p an empirical coefficient (triangular time base/lag), A the catchment area (km^2), L the length along the mainstream from outlet to the upstream divide (km), and L_c the length of the main stream from the outlet to a point on the stream nearest to the centroid of the watershed area (km). C_t is the coefficient for catchment gradient and storage. In the considered catchment, C_p = 0.51, C_t = 1.50, and L_c= 30 km. Due to the fact that the adopted rainfall time step T_R, equal to 1 day did not correspond to the condition $T_r= T_l/5.5$, where T_l the lag time, T_r an unit hydrograph duration, the time lag was adjusted to T_{lR} using the formula:

$$T_{lR} = T_l + \frac{T_R - T_r}{4} \tag{12}$$

...which resulted in T_{lR}=14.85 h.

Results and Discussion

Optimization of the Objective Function

Table 2 presents the estimators of the parameters: α_{min}, β_{min} for the beta SUH and k_{min}, and v_{min} for the Weibull SUH using the criterion min *F*.

The interpretation of the optimization of parameters α and β for the beta SUH is presented in Fig. 2 and for the Weibull SUH in Fig. 3. The figures are based on the wave from the winter half-year. Each graph focuses the variability of the objective function if one parameter varies and the second one is fixed as the optimal value. A clear decrease of the objective function *F* can be seen in the initial steps of the optimization until the function reaches a minimum,

Table 2. The parameters of the beta and Weibull distributions, which minimize *F*.

Type of the flood event	Beta SUH		Weibull SUH	
	α_{min}	β_{min}	k_{min}	v_{min}
Winter	2.78	3.38	2.50	0.53
Summer	7.56	10.84	3.90	0.43

which indicates the optimum value of the given parameter. Then the objective function increases. In the case of the parameter α and the beta SUH, we revaluated the peak discharge values in the unit hydrograph by moving both left and right to the min *F*.

Sensitivity Analysis of the Generated SUH

The power of the influence of the changes of parameters *α*, *β*, *k*, and *v* on the changes of the maximum discharges and on the times to peak for both SUHs in the winter half-year was expressed in the coefficients *S* and *e*.

The absolute sensitivity *S* was studied using (8) and the first partial derivatives, apart from the simple case $v \rightarrow f(mode)$ in (4) where the dependence is linear. The impact of each parameter was analyzed separately, while the second parameter was fixed. The results for *S* and for the peak discharge are presented in Table 3.

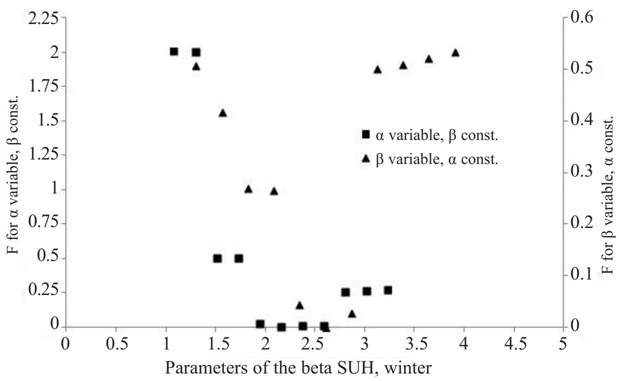

Fig. 2. Optimization of parameters of the beta SUH.

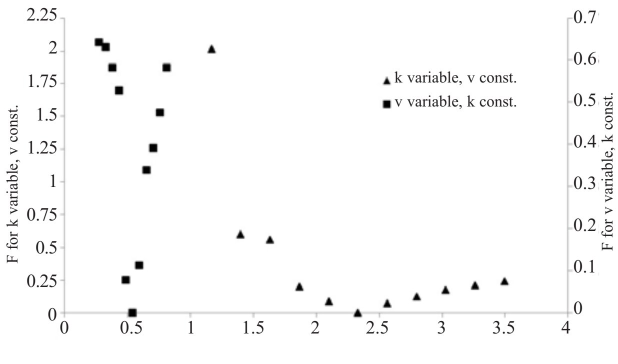

Fig. 3. Optimization of parameters of the Weibull SUH.

Table 3. The absolute sensitivity *S* of the peak discharge for the beta and Weibull SUH.

The parameter change step	Beta SUH		Weibull SUH	
	α=2.78, *β* variable	*β*=3.38, *α* variable	*k*=2.50, *v* variable	*v*=0.53, *k* variable
-0.2	-0.02	-0.01	0.59	-0.13
-0.1	-0.01	-0.01	0.22	-0.07
0.1	0.01	0.01	-0.17	0.06
0.2	0.02	0.01	-0.28	0.12

We deduce that:

- Both parameters have an analogous tendency: an increase of *α*, *β* causes an increase of the peak discharge and vice versa. The changes in the peak discharges in the beta SUH are proportional to changes of the parameters *α*, *β* in the considered intervals. The form of *f*(*mode*) in (2) implies that if both parameters are equal, they influence the peak discharge with the same power. However, as $\beta_{min} > \alpha_{min}$, the changes differ. The elasticity ratio *e* will indicate the stronger parameter in further analysis.
- An increase of *k*, *v* causes a decrease of the peak discharge and vice versa. The parameter *v* causes them to a greater degree than *k*. Analytically, the first partial derivative with respect to *v* equals:

$$\frac{\delta f(mode)}{\delta v} = -\frac{k}{v^2}\left(\frac{k-1}{k}\right)^{\frac{k-1}{k}} e^{-\frac{k-1}{k}} = -\frac{1}{v} f(mode)$$

is negative, which confirms an opposite response to changes and decreases quicker for $v<v_{min}$ than for $v>v_{min}$. That is why the absolute error is greater if this parameter is underestimated than otherwise.

The changes in the Weibull SUH are much stronger than in the beta, indicating the parameter *v* as dominant.

Figs. 4 and 5 display the graphs of the elasticity ratio *e*. The parameter change was from 0 to 25% in these figures.

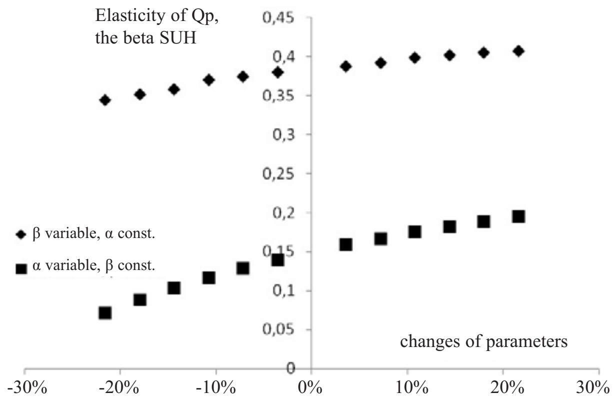

Fig. 4. The elasticity ratio for the maximum discharge, the beta SUH model.

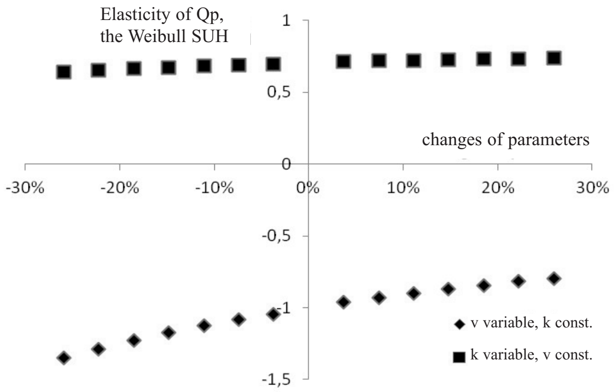

Fig. 5. The elasticity ratio for the maximum discharge, the Weibull SUH model.

The results confirm the analysis of S and additionally imply that:

- the peak discharge in the beta SUH is inflexible to changes of both α and β because the elasticity ratio is less than 1. However, β causes greater relative error than does α.
- the peak discharge in the Weibull SUH is inflexible to k and more flexible to v: its underestimation in comparison to the optimal value v_{min} causes overestimation of the peak discharge.

Subsequently, the influence of the parameters α, β, k, v on the time to peak was studied.

- The first partial derivative of the mode in the beta SUH with respect to parameter α equals:

$$\frac{\delta mode}{\delta\alpha} = \frac{\beta - 1}{(\alpha + \beta - 2)^2}$$

...and is positive for β>1. As we consider β from the neighbourhood of β_{min}=3.38 only, this implies that an overestimation of α causes an overestimation of the time to peak and vice versa. As $\frac{\delta mode}{\delta\alpha}(\alpha)$ is a decreasing function, the absolute error in the time to peak is greater for $\alpha<\alpha_{min}$ than for $\alpha>\alpha_{min}$. The first partial derivative with respect to parameter β equals:

$$\frac{\delta mode}{\delta\beta} = \frac{1 - \alpha}{(\alpha + \beta - 2)^2}$$

...and is negative for every α near 2.78. Hence an overestimation of β causes an underestimation of the time to peak, but the absolute error is greater for $\beta<\beta_{min}$ than for $\beta>\beta_{min}$. The power of the response of the time to peak to α changes is greater than to β changes, as $\left|\frac{\delta mode}{\delta\alpha}(\alpha_{min})\right| > \left|\frac{\delta mode}{\delta\beta}(\beta_{min})\right|$.

- The first partial derivative of the mode in the Weibull SUH with respect to parameter v equals:

$$\frac{\delta mode}{\delta v} = \left(\frac{k-1}{k}\right)^{\frac{1}{k}}$$

...and does not depend on v. This relates to the case when there are exactly two first non-zero components in the sum (7) and the further derivatives vanish. For each fixed k the error in v causes a proportional error in the time to peak, in the same direction. The derivative $\frac{\delta mode}{\delta v}$ was not calculated due to high complexity. However, the impact of parameter v on the mode was studied directly from the Weibull pdf.

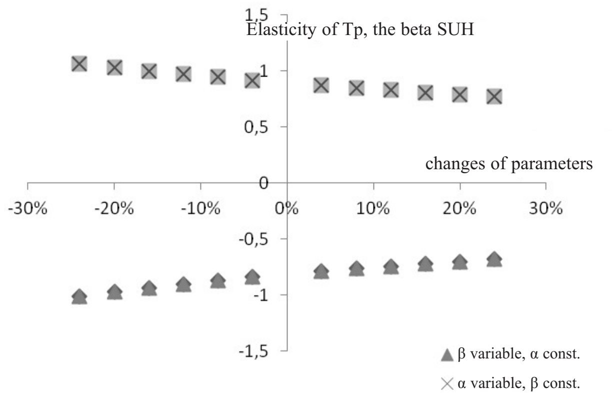

Fig. 6. The elasticity ratio for T_p, the beta SUH model.

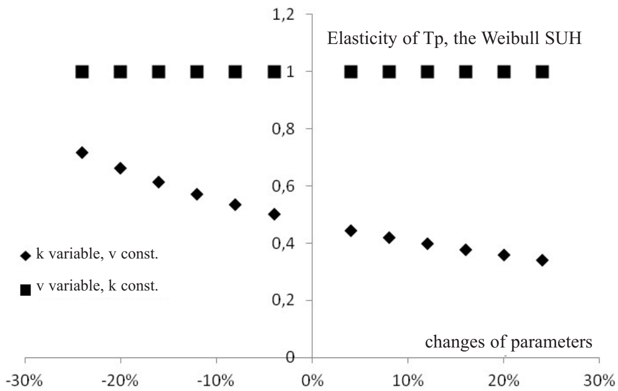

Fig. 7. The elasticity ratio for T_p, the Weibull SUH model.

The influence of the parameters α, β, k, v on the time to peak for both SUHs, expressed in the elasticity ratio e, is presented in Figs. 6 and 7.

The results confirm earlier results and imply that:

- the time to peak in the beta SUH is weak flexible to both parameters when they overestimate the optimal values and flexible if strong underestimation (20%) holds,
- the time to peak in the Weibull SUH is flexible to v (as mode is a linear function of v for Weibull distribution, see equation (4)) and inflexible to k.

Generally, the maximum discharge in the beta SUH model is less flexible to changes of parameters than in the Weibull SUH. The time to peak is flexible in a similar degree to the parameter v in the Weibull SUH and to both α and β in the beta SUH because all three parameters are featured by a similar elasticity ratio. The influence of k is weaker.

The above study was complemented for each SUH with numerical analysis of the simulated peak discharges for both parameters changing simultaneously. The results are presented in Table 4 and imply that:

Table 4. The simulated peak discharges in the beta and Weibull SUH for various parameters, the winter half-year.

	Beta SUH						Weibull SUH				
β/α	**2.58**	**2.68**	**2.78**	**2.88**	**2.98**	v/k	**2.1**	**2.3**	**2.5**	**2.7**	**2.9**
3.18	1.85	1.86	1.88	1.89	1.90	**0.33**	2.69	2.87	3.06	3.26	3.46
3.28	1.88	1.89	1.90	1.91	1.92	**0.43**	2.06	2.20	2.35	2.50	2.66
3.38	1.90	1.91	***1.92****	1.93	1.94	**0.53**	1.67	1.79	***1.92****	2.03	2.15
3.48	1.93	1.93	1.94	1.95	1.96	**0.63**	1.41	1.50	1.60	1.71	1.81
3.58	1.95	1.96	1.96	1.97	1.98	**0.73**	1.21	1.30	1.38	1.47	1.56

*related to the peak discharge (Q_p=1.92 m³/s) for optimal parameters

Table 5. The validation of the beta and Weibull SUHs and of the Snyder UH, the winter half-year.

Unit hydrograph	Error of Q_p (to the Snyder's model) [%]	V [10^6 m³]	Error of V (to the observed) [%]	Efficiency E [%]
Observed wave		8.43		
Beta SUH	-2.48	9.42	11.74	65.1
Weibull SUH	-2.48	9.10	7.94	71.7
Snyder UH		7.94	-5.81	89.3

*the error of the volume was calculated as $\frac{V_{sim}-V_{obs}}{V_{obs}}$, analogously for the error of Q_p.

1. An overestimation of both parameters (especially β) in comparison to the optimal values α_{min}, β_{min} in the beta SUH causes overestimation of the simulated peak discharge. The response of the model is contrary for both parameters underestimated.
2. The peak discharge in the Weibull SUH is overestimated if the parameter v is lower than the optimal value ($v<v_{min}$). If $v>v_{min}$ then it is underestimated. These properties hold for every k. The parameter k has weaker influence on the simulated peak discharge. The strongest overestimation was achieved for $k>k_{min}$ and $v<v_{min}$ and the strongest underestimation otherwise.

Comparison and Validation of the Models

Figs. 8 and 9 show the verification of the prepared synthetic hydrographs using the SUH beta and Weibull method. The total recorded rainfall depth that caused the analyzed high water level was 42.10 mm and lasted 4 days. The results of the detailed comparison between the SUH hydrographs (beta, Weibull and Snyder) are presented in Table 5.

The corollaries are as follows:

1. Snyder's model overestimated the peak discharges in SUHs.
2. The total volume was overestimated by both SUHs and the errors were greater than using Snyder's model.
3. The beta and Weibull SUHs are featured by good efficiency which is, however, lower than in the Snyder's unit hydrograph model that represents very good efficiency.

Generally, the beta and Weibull SUHs correctly describe the actual flood events and both models are good, indicating the Weibull SUH as more efficient. Generally, both constructed SUHs have the role to simulate perfectly the peak discharge while the Snyder's model better estimates total volume.

Conclusions

The applied method correctly described the precipitation flood in the catchment of the Jasiołka River. The proposed SUHs were characterized by good efficiency in determining hypothetical hydrographs, and the usefulness of the applied objective function in the medium size mountain catchment was confirmed.

Our paper evaluates the sensitivity of both SUH models to parameter changes. The response of the model is reflected in absolute and relative sensitivities. They were investigated using the first partial derivatives and/or numerical calculations. The peak discharge in the beta SUH is less sensitive than in the Weibull SUH: the influence of each parameter of the Weibull pdf is greater than of these in the beta pdf. The influence of the parameters v, α, and β on the time to peak is similar. The Weibull SUH is most sensitive to changes in parameter v than in k. The analysis was complemented with the study of the simulated peak discharge when both parameters change simultaneously. Hereby, the deeper insight in the nature of the changes was performed and the earlier results were confirmed.

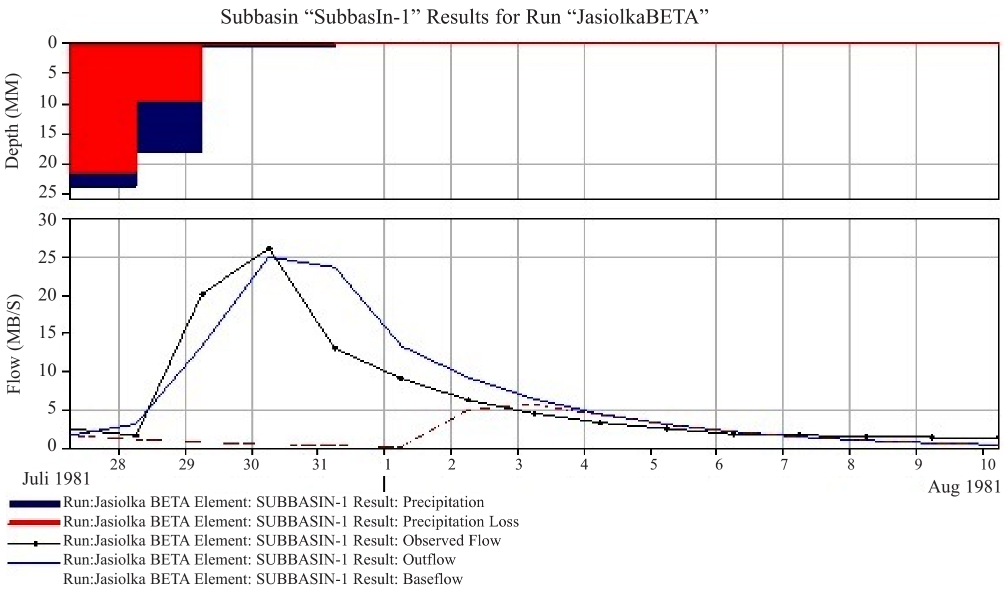

Fig. 8. Verification of the SUH model based on the beta distributions for the summer half-year, the Jasiołka River at Jasło.

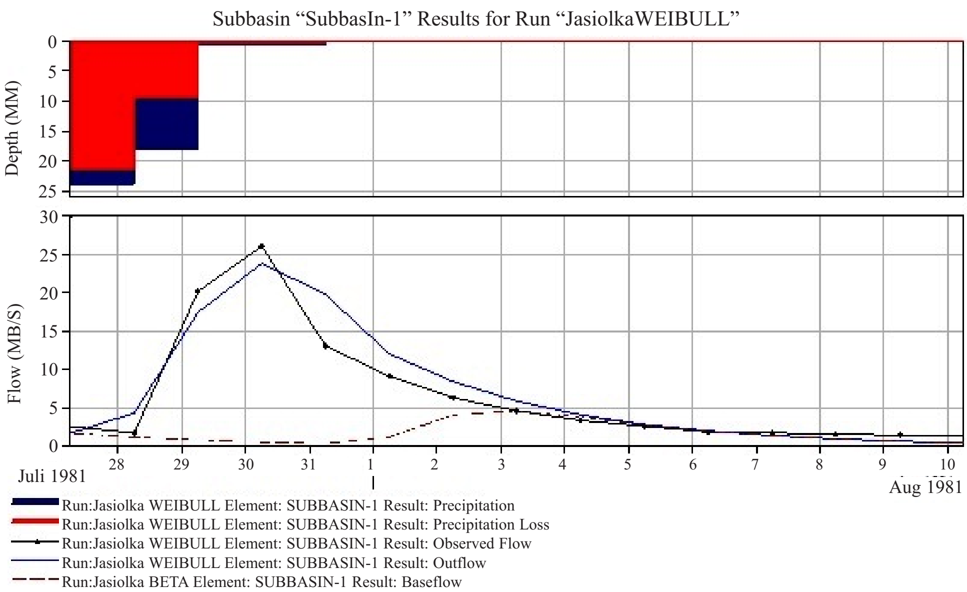

Fig. 9. Verification of the SUH model based on the Weibull distributions for the summer half-year, the Jasiołka River at Jasło.

References

1. VAN ALPHEN J., MARTIINI F., LOAT R., SLOMP R. PASSCHIER R. Flood risk mapping in Europe, experiences and best practices. J. of Flood Risk Management **2**, 285, **2009**.
2. GUIDELINES FOR ELABORATE OF FLOOD HAZARD MAP. National Water Management Authority, Warsaw, Poland. **2009**.
3. GUIDELINES FOR ELABORATE OF FLOOD RISK MAP. National Water Management Authority, Warsaw, Poland. **2009**.
4. EBRAHIMIAN M., NURUDDIN A.A.B., SOOM M.A.B.M., SOOD A.M., NENG L.J. Runoff estimation in steep slope watershed with standard and slope-adjustment Curve Number Method. Pol. J. Environ. Stud., **21**, (5), 1191, **2012**.
5. KANOWNIK W., KOWALIK T., BOGDAŁ A., OSTROWSKI K. Quality Categories of Stream Water Included in a Small Retention Program. Pol. J. Environ. Stud., **22**, (1), 159, **2013**.
6. BOGDAŁ A., OSTROWSKI K. Water outflows from small agricultural catchment in Wilamowickie Foorhills. Acta Sci. Pol., Formatio Circumiectus, **7**, (3), 13, **2008**.
7. PEARSON F.H. Unit hydrograph by diffusion analogy. J. Hydrol. **9**, (2), 299, **1970**.
8. PONCE V.M. Engineering Hydrology: Principles and Practices. Prentice Hall, Upper Saddle River, New Jersey, **1989**.
9. PONCE V.M., CHAGANTI P.V. Variable-parameter Muskingum-Cunge method revisited. J. Hydrol. **162**, 433, **1994**.
10. BHUNYA P.K., BERNDTSSON R., OJHA C.S., MISHRA S.K. Suitability of gamma, Chi-square, Weibull and beta distributions as synthetic unit hydrographs. J. Hydrol. **334**, 28, **2007**.

11. BEVEN K.J. Rainfall-Runoff modelling. The Primer, Wiley & Sons, West Sussex, England, **2001**.
12. SALAMI A.W., BILEWU S.O., AYANSHOLA A.M., DORITOLA S.F. Evaluation of synthetic unit hydrograph methods for the development of design storm hydrographs for Rivers in South-West, Nigeria. J. of American Science **5**, (4), 23, **2009**.
13. BELETE M.A. Synthetic Unit Hydrographs in the Upper Awash and Tekeze Basins. Methods, Procedures and models.VDM Verlag Dr Müller, **2009**.
14. RAMIREZ J.A. Prediction and Modeling of Flood Hydrology and Hydraulics, in: Wohl, E., (Eds.), Chapter 11 of Inland Flood Hazards: Human, Riparian and Aquatic Communities. Cambridge University Press, **2000**.
15. MAIDMENT D.R. Handbook of Hydrology. McGraw-Hill. **1993**.
16. VEN TE CHOW, MAIDMENT D.K., MAYS L.W. Applied Hydrology. McGRAW-HILL BOOK COMPANY, New York. **1988**.
17. NOORBAKHSH M.E., RAHNAMA M.B., OGUNLELA S.M. Estimation of Instantaneous Unit Hydrograph with Clark's Method Using GIS Techniques. J. of Applied Science **5**, (3), 455, **2005**.
18. OGUNLELA A.O., KASALI M.Y. Evaluation of four methods of storm hydrograph development for an ungaged watershed. Nigerian Journal of Technological Development. Faculty of Engineering and Technology, University of Ilorin, Ilorin, Nigeria **2**, 25, **2002**.
19. BHUNYA P.K., MISHRA S.K., ANDBERNDTSSON R. Simplified two-parameter Gamma distribution for derivation od Synthetic Unit Hydrograph. Journal of Hydrologic Engineering **7-8**, 226, **2003**.
20. WAŁĘGA A. Impact of flood wave characteristics and catchments on the parameters of the synthetic unit hydrographs – beta and Weibull distribution. Infrastructure and Ecology of Rural Areas **7**, 29, **2011**.
21. LEE M.T., BLANK D., DELLEUR J.W. A program for estimating runoff from Indiana watershed, Part II. Assembly of hydrologic and geomorphologic data for small watersheds in Indiana, Tech. Rep. No.23. Purdue University Water Resources Research Center, Lafayette, **1972**.
22. AL-WAGDANY A.S., RAO A.R. Estimation of the velocity parameter of the geomorphologic instantaneous unit hydrograph. Water Resour. Manag. **11**, (1), 1, **1997**.
23. AHMAD M.M., GHUMMAN A.R., AHMAD S. Estimation of Clark's Instantaneous Unit Hydrograph Parameters and Development of Direct Surface Runoff Hydrograph. Water Resour. Manag. **23**, 2417, **2009**.
24. MCCUEN R.H. Modeling Hydrologic Change, Statistical Methods, Lewis Publishers, a CRC Press Company, **2003**.
25. MAIDMEND D.R., HOOGERWERF T.N. Parameter Sensitivity in Hydrologic Modeling. Technical Report. The University of Texas at Austin, **2002**.
26. NASH J.E., SUTCLIFFE J.V. River flow forecasting through conceptual models, Part-I: a discussion of principles. J. Hydrol. **10**, (3), 282, **1970**.
27. WĘGLARCZYK S. The interdependence and applicability of some statistical quality measures for hydrological models. J. Hydrol. **206**, 98, **1998**.
28. MORIASI D.N., ARNOLD J.G., VAN LIEW M.W., BINGNER R.L., HARMEL R.D., VEITH T.L. Model evaluation guidelines for systematic quantification of accuracy in watershed simulations. American Society of Agricultural and Biological Engineers. ISSN 0001-2351, **50**, (3), 885, **2007**.
29. HYDROLOGIC MODELLING SYSTEM HEC-HMS. User's Manual. US Army Corps of Engineers. Hydrologic Engineering Center, **2009**.
30. BANASIK K., NGOC PHAM. Modelling of the effects of land use changes on flood hydrograph in a small catchment of the Płaskowicka, southern part of Warsaw, Poland. Annals of Warsaw University of Life Sciences – SGGW, Land Reclamation, **42**, (2), 229, **2010**.